## **Technical Notes**

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### Aerothermodynamic Characteristics of a Resonance Tube Driven by a Subsonic Jet

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#### Introduction

RESONANCE tube, in its simplest form, is a cylindrical or rectangular tube closed at one end and facing a coaxial fluid jet at the open end. If appropriate geometric and fluid conditions are satisfied by the jet/tube combination, the fluid within the tube is excited to violent harmonic motion. If the tube is sufficiently long, the intense periodic oscillations give rise to strong thermal effects which result in resonance tube endwall temperatures significantly higher than the corresponding jet stagnation temperatures. For subsonic and low Mach number supersonic jets, the fundamental resonance frequency is independent of the jet Mach number  $M_{\rm jet}$  and can be approximated by f = C/4L, where C is the speed of sound of the jet and L is the length of the resonance tube.

Previous experimental and analytical work, reviewed by Przirembel and Fletcher, was concerned primarily with resonance tubes driven by supersonic and underexpanded, choked jets. For subsonic jets, only very limited experimental data have been reported. Savory<sup>2</sup> has observed fluid oscillations for subsonic jets, when the resonance tube is supported axially by a cylindrical rod passing through the nozzle. The fluid dynamics of this flowfield have been analyzed by Broucher, Maresca, and Bournay. 3 Sprenger 4 was able to obtain oscillating flows with jet Mach numbers as low as 0.52 by placing a thin thread across the jet axis. Vrebalovich<sup>5</sup> observed oscillations in a cavity placed in a subsonic flowfield by locating a ring trip upstream of the tube mouth. More recently Przirembel, Fletcher, and Wolf<sup>6</sup> have observed resonance in two-dimensional resonance tubes driven by a subsonic, two-dimensional jet, without the introduction of any tripping device.

#### **Experimental Program**

A simple schematic of the axisymmetric nozzle/resonance tube model and pertinent geometrical parameters are shown in Fig. 1. The air jet was generated by a simple, converging nozzle with a contraction ratio of 3 to 1. The nozzle exit diameter was 0.375 in. (0.953 cm).

The resonance tube had an inside diameter d of 0.375 in. (0.953 cm). The length L to diameter d ratio was 30 for all

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tests. The resonance tube endwall was designed and constructed to accommodate iron-constantan thermocouples for temperature measurements, and an orifice for pressure measurements.

#### **Results and Discussion**

The variable test parameters were the ratio of the separation distance between the nozzle exit and the resonance tube inlet to the nozzle exit diameter S/D, the ratio of the jet stagnation pressure to the ambient pressure  $P_0/P_a$ , the ratio of the diameter of the trip to the nozzle exit diameter b/D, and the separation distance of the trip wire from the nozzle exit x. The nominal jet stagnation temperature was the atmospheric temperature.

Of principal interest in this experimental investigation was the determination of the temperature and thermal effects in the vicinity of the resonance tube endwall. Figures 2 and 3 summarize some typical results for jet flows with and without a trip. From Fig. 2, it can be seen that there is no measurable increase in the endwall temperature above the jet stagnation temperature for a simple subsonic jet  $(M_{\text{jet}} = 0.73)$ . However, as a thin wire is placed across the center of the jet exit plane, there is a significant temperature increase. As the jet Mach number is increased in Fig. 3, there is some limited resonance tube heating without the trip. The shape of curves for flow in the presence of the trip are quite similar. The peak endwall temperature increases as the jet Mach number increases. These results are consistent with those reported by Sprenger.<sup>4</sup> Broucher et al.<sup>3</sup> have argued that the presence of the trip results in a stagnation pressure deficiency in the jet, which

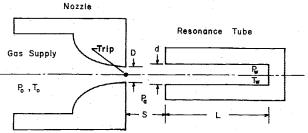


Fig. 1 Schematic of resonance tube model.

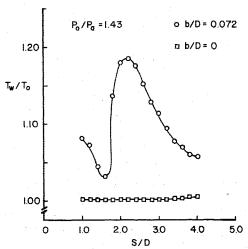


Fig. 2 Endwall temperature variation with S/D for  $P_{\theta}/P_{\alpha} = 1.43$ .

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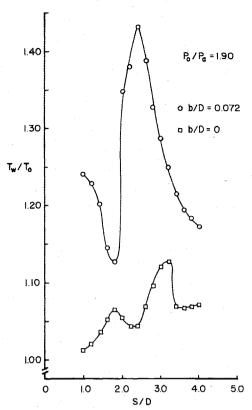


Fig. 3 Endwall temperature variation with S/D for  $P_{\theta}/P_{\alpha} = 1.90$ .

allows the resonance tube outflow to penetrate the jet. From wake calculations for the trip immersed in the core of the jet, it can be shown that the maximum velocity defect is on the order of 20% for the maximum endwall temperatures. At the inlet plane for the resonance tube, the trip wake width is on the order of one-half the resonance tube diameter. By comparing the frequency of oscillation (286 Hz) and the shedding frequency (73,400 Hz) associated with the trip, there is no indication that the resonance condition is driven by the wake shedding process. The endwall pressure measurements showed a significant correlation with the endwall temperature curves, i.e., pressure increases occurred at the same S/D values as temperature increases.

The effect of the relative location of the trip wire x is shown in Fig. 4. As can be seen, there is a significant decrease in the maximum endwall temperature, as well as a shift in S/D as the wire is moved away from the jet exit plane. The jet centerline velocity defect increases very slightly at the resonance tube inlet. However, the mass flow of the jet entering the resonance tube decreases.

In order to determine the influence of trip diameter on the maximum endwall temperature, a series of experiments were performed which resulted in curves for different trip diameters similar to those shown in Figs. 2-4. For these experiments, the trips were placed at the exit plane of the nozzle. A summary of these results is presented in Fig. 5, where the maximum endwall temperatures are plotted for various trip diameters. The primary observation is that there is no distinct peak for a particular combination of wire diameter and jet velocity. Vortex shedding frequencies based on a Strouhal number of 0.21 do not provide any additional information on the mechanism of resonance since the estimated peaks appear independent of jet velocity. It should be noted that the calculated shedding frequencies are on the order of 200 times the frequency of oscillation in the resonance tube. From these observations it can be concluded that the oscillations in the resonance tube are not directly driven by the vortices shed from the trip.

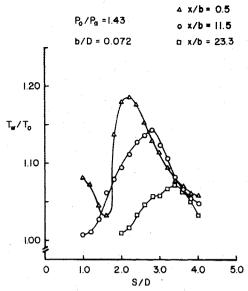


Fig. 4 Endwall temperature variation with S/D and relative location of trip for  $P_\theta/P_a=1.43$ .

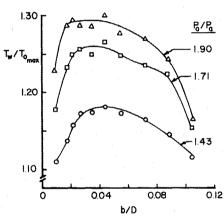


Fig. 5 Maximum endwall temperature variation with trip diameter.

As indicated earlier, Broucher et al. 3 have suggested that the best method to obtain oscillations in a resonance tube driven by a subsonic jet is to decrease the kinetic energy of the jet in the neighborhood of the axis in order to act upon the radial pressure distribution at the resonance tube entrance. Without detailed jet velocity and static pressure measurements near the resonance tube, it is not clear at this point if the present experimental results support their analysis. The wake defect of the present cylinder is still significant for the separation distances at which maximum endwall temperatures were measured. However, it is also possible that the trip wake vortices cause disturbances in the outer region of the jet and thereby initiate instabilities which ultimately lead to oscillations in the resonance tube.

#### Acknowledgment

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# Two-Parameter Skin-Friction Formula for Adiabatic Compressible Flow

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#### Introduction

THE derivation of a skin-friction formula depends on the THE derivation of a skin-friction results are similarity of velocity profiles. If the velocity profiles are similar from the wall to the edge of the boundary layer, a oneparameter skin-friction formula, expressing the relation between skin-friction coefficient  $C_f$  and Reynolds number Re, will be sufficient, e.g., those developed by Prandtl, Schlichting, von Karman, Schoenherr, etc. A one-parameter skin-friction formula is adequate to describe flat-plate boundary-layer flow where velocity profiles do collapse. However, for flows with pressure gradient or history effect, this is no longer valid. If the similarity breaks down everywhere within the boundary layer, then a correlated skin friction can be obtained by Clauser's 1 plot for the inner region of the velocity profile. If a more general formula which involves Reynolds number in term of freestream velocity is needed, some fairing of velocity profile to account for the nonsimilarity in the outer region must be included. The fairing can be done by using Coles' wake function or Sarnecki's  $^{3,4}$  intermittency function which give the  $C_f$  indirectly, or by Ludwieg and Tillmann's 5 method which vields an explicit formula for  $C_f$ . The fairing of velocity profile introduces yet another variable besides Re, i.e. wake parameter II in Coles' method or shape factor H in Ludwieg and Tillmann's method. Hence, a two-parameter skin-friction formula is needed for flows with pressure gradient or history effect.

For compressible flow, the influence of Mach number has to be accounted for. Several authors (Winter and Gaudet, <sup>6</sup> Spalding and Chi, <sup>7</sup> Sommer and Short <sup>8</sup>) have attempted to include the Mach number effect as compressibility factors for  $C_f$  and  $Re_\theta$  so that a one-parameter skin-friction formula for flat-plate flow can be derived empirically as in incompressible flow. In this paper, a two-parameter skin-friction formula for compressible adiabatic flow with pressure gradient or history effect will be developed.

#### Experiment

The experiment was carried out in a 114×165 mm blowdown-type wind tunnel at a Mach number of 2.5 to study the relaxing boundary layer after it had been disturbed by an oblique incident shock and Prandtl-Meyer expansion. The adiabatic, turbulent boundary layer of about 6-mm thickness on an expansion-corner plate which had a sharp 6-deg turning angle was impinged by a plane oblique shock with flow

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deflection angle ( $\alpha$ ) of 4, 6, or 8 deg. The streamwise shock impingement position  $x_{sh}$  measured from the expansion corner can be varied from -60 mm to 60 mm. The boundary-layer Mach number profiles were measured by Pitot tube and the velocity profiles were derived by assuming a quadratic temperature relation. The skin-friction distribution was measured by Preston tube and Allen's  $^{10}$  calibration curve was used for data reduction.

#### Formulation

As mentioned before, the derivation of a skin-friction formula depends on wall similarity. The mixing-length theory and logarithmic law of the wall are mutually exclusive for flows with pressure gradient or history effect. The mixing-length theory  $^{11}$  predicts no wall similarity. However, for incompressible flow near separation, Simpson et al.  $^{12}$  have shown that the law of the wall still exists. The present data for compressible flow just downstream of a separation region also support the law of the wall when plotted in Van Driest's  $^{13}$  transformed velocity, (Fig. 1),  $U^*$  where

$$U^* = \int_0^U \left(\frac{\rho}{\rho_w}\right)^{1/2} \mathrm{d}U$$

In this figure, the velocity profiles of this relaxing boundary layer at five x locations measured from the expansion corner are plotted on staggered ordinates. The dash lines are Coles' wake function with  $\Pi$  chosen to fit the data at the edge of the boundary layer. In view of the presence of a law of the wall, a skin-friction formula can be developed.

The two-parameter skin-friction formula was developed along similar lines to that of Ludwieg and Tillmann's<sup>5</sup> incompressible formula. This is done by approximating the transformed inner velocity profile by a power law as

$$\frac{U^*}{U_T} = f\left(\frac{U_\tau y}{\nu_w}\right) \tag{1}$$

which at  $y = \theta^*$ , where  $\theta^*$  is the momentum thickness based on transformed velocity defined as

$$\theta^* = \int_0^\infty \frac{U^*}{U^*} \left( 1 - \frac{U^*}{U^*} \right) \mathrm{d}y$$

becomes

$$\frac{U_{\tau}}{U_{\theta^*}^*} = h\left(\frac{U_{\theta^*}^* \theta^*}{\nu_{w}}\right) \tag{2}$$

Equation (2) is only valid if the wall similarity applies up to  $y=\theta^*$ . Thus, a profile parameter  $\gamma$  which is equal to the value or projected value of  $U^*_{\theta^*}/U_e$  at  $y=\theta^*$  when Eq. (2) is valid, has to be introduced to account for the departure. By introducing

$$\gamma = \frac{U^*_{\theta^*}}{U_e}, \quad U_{\tau} = \sqrt{\frac{\tau_w}{\rho_w}} = U_e \left(\frac{\rho_e}{\rho_w}\right)^{\frac{1}{2}} \sqrt{\frac{C_f}{2}}, \quad Re_{\theta^*} = \frac{U_e \theta^*}{\nu_w}$$

Eq. (2) becomes

$$C_f = \gamma^2 \left( \rho_w / \rho_e \right) G(Re_{\theta^*} \cdot \gamma) \tag{3}$$

where  $G = 2h^2$ .

The function G can be derived by considering adiabatic compressible flows without pressure gradient. In this case,  $C_f$  is a function of  $Re_{\theta}$  and M, and Eq. (3) becomes

$$F(Re_{\theta^*}, M) = \gamma_0^2 (\rho_w/\rho_e)_0 G(Re_{\theta^*} \cdot \gamma_0)$$
 (4)

where subscript 0 denotes values at dP/dx = 0,  $\gamma_{\theta}$  depends on the shape of the velocity profile and is therefore a weak